

8300

GCSE MATHEMATICS

Foundation and Higher tier – Questions by Topic – Mark Scheme

Geometry

Volume – Finding the volume of prisms

v1.0

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt

M	Method marks are awarded for a correct method which could lead to a correct answer.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied
B	Marks awarded independent of method.
ft	Follow through marks. Marks awarded for correct working following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
M dep	A method mark dependent on a previous method mark being awarded.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
oe	Or equivalent. Accept answers that are equivalent. e.g. accept 0.5 as well as $\frac{1}{2}$
[a, b]	Accept values between a and b inclusive.
3.14...	Allow answers which begin 3.14 e.g. 3.14, 3.142, 3.1416
Use of brackets	It is not necessary to see the bracketed work to award the marks.

Examiners should consistently apply the following principles

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a student has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the student. In cases where there is no doubt that the answer has come from incorrect working then the student should be penalised.

Questions which ask students to show working

Instructions on marking will be given but usually marks are not awarded to students who show no working.

Questions which do not ask students to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Students often copy values from a question incorrectly. If the examiner thinks that the student has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Q1.

$$\frac{1}{2} \times 10 \times 5 \times 12$$

oe

300

cm³

M1

A1

B1

[3]

Q2.

$$\frac{1}{2} \times 9 \times 5 \times 10$$

225

cm³

M1

A1

B1

[3]

Q3.

$$0.5 \times 4 \times 6 (= 12)$$

Their 12×16

$6 \times 4 \times 16 \div 2$ is M2

192

cm³

M1

M1Dep

A1

B1

[4]

Q4.

(a) $\frac{1}{2} (6.5 + 8.3) 3.2$

23.68 or 23.7

(b) their 23.68×200

4736 or 4740

M1

A1

M1

A1 ft

[4]

Q5.

(a) 8

B1

(b)

$$3500 \div 20 \text{ or } \frac{3500}{20}$$

oe eg $350 \div 2$

M1

175

SC1 digits 175

A1

Additional Guidance

Ignore units

[3]

Q6.

Alternative method 1

$$8 \div 4 \text{ or } 2$$

$$4 \times 2 = 8$$

or implies volume = $4 \times \text{area of triangle}$

B1

$$(\text{Area of triangle} =) \frac{1}{2} \times x \times x \times \sin 60$$

$$\text{or } \frac{1}{2} \times x \times x \times \frac{\sqrt{3}}{2}$$

oe

B1

$$\frac{1}{2} \times x \times x \times \sin 60 = 2$$

$$\text{or } \frac{1}{2} \times x \times x \times \frac{\sqrt{3}}{2} = 2$$

$$\text{or } (x^2 =) \frac{4}{\sin 60} \text{ or } 4.59... \text{ or } 4.6...$$

$$\text{or } (x^2 =) \frac{8}{\sqrt{3}}$$

$$\cos 30 = \frac{h}{2.149} \text{ or } \sin 60 = \frac{h}{2.149}$$

$$\text{or } 2 = \frac{1}{2} \times 2.149 \times h$$

$$\text{or } h^2 = \frac{6}{\sqrt{3}} \text{ or } 2\sqrt{3}$$

oe

M1

$$(h =) [1.81, 1.87]$$

$$\text{or } (x =) [2.1, 2.15]$$

$$\text{or } (x^2 =) [4.59, 4.66]$$

oe

A1

[1.81, 1.87] and No

A1

Alternative method 2

$8 \div 4$ or 2

$$4 \times 2 = 8$$

or implies volume = $4 \times$ area of triangle

B1

(half the base =) $h \tan 30$

oe

B1

$$h \tan 30 \times h = 2$$

M1

$$(h^2 =) [3.46, 3.47]$$

$$\text{or } (h =) [1.81, 1.87]$$

A1

[1.81, 1.87] and No

A1**Alternative method 3**

$8 \div 4$ or 2

$$4 \times 2 = 8$$

or implies volume = $4 \times$ area of triangle

oe

B1

$$\tan 60 = \frac{1.95}{\text{half the base}}$$

$$\text{or } \tan 30 = \frac{\text{half the base}}{1.95}$$

$$\text{or (half the base =) } \frac{1.95}{\tan 60}$$

$$\text{or (half the base) = } 1.95 \times \tan 30$$

or 1.125... or 1.13

$$\text{or } \frac{13\sqrt{3}}{20}$$

B1

their $1.125... \times 1.95$

or their $1.125... \times 1.95 \times 4$

M1

oe

(Area of triangle =) [2.19, 2.2]

or

(Volume of prism =) [8.7, 8.8]

oe

A1

[2.19, 2.2] and No

or

[8.7, 8.8] and No

A1

Alternative method 4 $8 \div 4$ or 2

$4 \times 2 = 8$

*or implies volume = $4 \times$ area of triangle***B1**

$$x^2 = h^2 + \left(\frac{x}{2}\right)^2$$

$$\text{or } h^2 = x^2 - \left(\frac{x}{2}\right)^2$$

$$\text{or } h^2 = \frac{3}{4} x^2$$

$$\text{or } h = \frac{\sqrt{3}}{2} x$$

$$\text{or } \frac{1}{2} xh = 2$$

oe

B1

$$\frac{1}{2} x \times \frac{\sqrt{3}}{2} x = 2$$

$$\text{or } \frac{1}{2} \times \frac{2}{\sqrt{3}} h \times h = 2$$

$$\text{or } h^2 = \frac{8}{\sqrt{3}} - \frac{2}{\sqrt{3}} \text{ or } \frac{6}{\sqrt{3}} \text{ or } 2\sqrt{3}$$

$$\text{or } h^2 = 2.149^2 - \left(\frac{2.149}{2}\right)^2$$

oe

M1

$$(h^2) [3.46, 3.47]$$

$$\text{or } (h) [1.81, 1.87]$$

A1

[1.81, 1.87] and No

A1

Additional Guidance

Throughout mark scheme:

x represents the length of one side of the triangle

h represents the perpendicular height of the triangle

The principle of this mark scheme is as follows

Fact

B1

Different correct fact

B1

Any correct equation set up involving only one variable (need not be simplified)

M1

Any answer in range

A1

An answer in range giving the full solution with the correct conclusion

A1

$$\frac{1}{2} ab \sin C = 2$$

(given on the formula sheet)

B1B0

[5]

Q7.

Alternative method 1: works out a scale factor

$$\frac{1}{2} \times 3(L) \times 4(L) \times 12(L) \text{ or } 72(L^3)$$

where L is any variable or any positive value

oe volume

$$\text{eg } (L = 2) \frac{1}{2} \times 6 \times 8 \times 24 \text{ or } 576$$

M1

1125 ÷ their 72

$$\text{or } \frac{125}{8} \text{ or } 15.625$$

oe eg $1125 \times 2 \div (3 \times 4 \times 12)$

$$\text{eg } (L = 2) 1125 \div \text{their } 576 \text{ or } \frac{125}{64}$$

M1dep

$$\sqrt[3]{\text{their } \frac{125}{8}} \text{ or } \frac{5}{2} \text{ or } 2.5$$

oe

$$\text{eg } (L = 2) \sqrt[3]{\text{their } \frac{125}{64}} \text{ or } \frac{5}{4} \text{ or } 1.25$$

M1dep

$$2 \times 3 \times \text{their } 2.5 + 2 \times 4 \times \text{their } 2.5 + 2 \times 5 \times \text{their } 2.5 + 3 \times 12 \times \text{their } 2.5$$

oe

eg $(L = 2)$

$$2 \times 6 \times \text{their } 1.25 + 2 \times 8 \times \text{their } 1.25 + 2 \times 10 \times \text{their } 1.25 + 3 \times 24 \times \text{their } 1.25$$

M1dep

150

SC4 [119, 119.1]

A1

Alternative method 2: works out a value of a , b , c or d Correct expression for volume in terms of a or b

eg $\frac{1}{2} \times a \times \frac{4a}{3} \times \frac{12a}{3}$ or $\frac{8a^3}{3}$

or

$\frac{1}{2} \times \frac{3b}{4} \times b \times \frac{12b}{4}$ or $\frac{9b^3}{8}$

oe in terms of c or d

eg $\frac{1}{2} \times \frac{3c}{5} \times \frac{4c}{5} \times \frac{12c}{5}$ or $\frac{72c^3}{125}$

or

$\frac{1}{2} \times \frac{3d}{12} \times \frac{4d}{12} \times d$ or $\frac{d^3}{24}$

may be implied by an equation

eg $a \times \frac{4a}{3} \times \frac{12a}{3} = 1125 \times 2$

M1

$a^3 = 1125 \div \text{their } \frac{8}{3}$ or $a^3 = \frac{3375}{8}$

or

$b^3 = 1125 \div \text{their } \frac{9}{8}$ or $b^3 = 1000$

oe

eg $c^3 = 1125 \div \text{their } \frac{72}{125}$ or $c^3 = \frac{15\,625}{8}$

or

$d^3 = 1125 \div \text{their } \frac{1}{24}$ or $d^3 = 27\,000$

M1dep

$a = \sqrt[3]{\text{their } \frac{3375}{8}}$ or $a = 7.5$

or

$b = \sqrt[3]{1000}$ or $b = 10$

oe

eg $c = \sqrt[3]{\text{their } \frac{15\,625}{8}}$ or $c = 12.5$

or

$d = \sqrt[3]{27\,000}$ or $d = 30$

M1dep

$2 \times \text{their } a + 2 \times \frac{4}{3} \times \text{their } a + 2 \times \frac{5}{3} \times \text{their } a + 3 \times \frac{12}{3} \times \text{their } a$

or

$2 \times \frac{3}{4} \times \text{their } b + 2 \times \text{their } b + 2 \times \frac{5}{4} \times \text{their } b + 3 \times 3 \times \text{their } b$

oe correct method using their c or their d **M1dep**

150

SC4 [119, 119.1]

A1**Additional Guidance**

Up to M3 may be awarded for correct work with no answer or incorrect answer, even if this is seen amongst multiple attempts

[5]

Q8.

Alternative method 1

$$5^2 + 12^2 \text{ or } 169$$

$$\text{or } \sqrt{5^2 + 12^2} \text{ or } 13$$

oe

M1

$$\sqrt{16^2 - \text{their } 169}$$

or

$$\sqrt{16^2 - \text{their } 13^2}$$

$$\text{or } \sqrt{87} \text{ or } [9.3, 9.33]$$

$$\text{oe eg } \sqrt{16^2 - 5^2 - 12^2}$$

may be implied eg [18.6, 18.7]

M1dep

$$0.5 \times 5 \times 12 \times 2 \times \text{their } [9.3, 9.33]$$

oe

M1dep

$$[558, 559.8] \text{ or } 60 \sqrt{87}$$

accept 560 with full method seen

$$\text{SC3 } [1116, 1119.6] \text{ or } 120 \sqrt{87}$$

A1

Alternative method 2

$$16^2 - 5^2 \text{ or } 231$$

$$\text{or } \sqrt{16^2 - 5^2}$$

$$\text{or } 15.19(8\dots) \text{ or } 15.199 \text{ or } 15.2$$

oe

M1

$$\sqrt{\text{their } 231 - 12^2}$$

or

$$\sqrt{\text{their } 15.2^2 - 12^2}$$

$$\text{or } \sqrt{87} \text{ or } [9.3, 9.33]$$

$$\text{oe eg } \sqrt{16^2 - 5^2 - 12^2}$$

may be implied eg [18.6, 18.7]

M1dep

$$0.5 \times 5 \times 12 \times 2 \times \text{their } [9.3, 9.33]$$

oe

M1dep

$$[558, 559.8] \text{ or } 60 \sqrt{87}$$

accept 560 with full method seen

$$\text{SC3 } [1116, 1119.6] \text{ or } 120 \sqrt{87}$$

A1

Additional Guidance

Lengths may be seen on the diagram

1st and 2nd M marks can be awarded even if not subsequently used

$$5^2 + 12^2 + 16^2$$

M1M0M0A0**[4]**